Abstract

We report measured temperatures inside the single polymer fuel cell, and thermal conductivities and heat transfer coefficients calculated from these. Temperatures were measured next to the membrane on its two sides, and in the gas channels. Higher temperatures (5°C or more at 1 A/cm²) were found at the membrane electrode surface than in the gas channels. The thermal conductivity of the membrane (λₘ) was small, as expected from the properties of water and polymer, while the heat transfer coefficient of the electrode surfaces (λₛ) was smaller, 1000 ± 300 W/m² K for a layer thickness of 10 µm. The real coefficient is smaller, since the measured temperatures are systematically smaller than the real ones. The electrode surface heat transfer coefficient is not previously reported. The average value for the catalyst surface plus gas diffusion layer was 0.2 W/m² K.

Keywords: Heat production; Irreversible thermodynamics; Fuel cell; Surface heat transfer coefficient; Thermal conductivity

1. Introduction

Knowledge of the heat production and heat transport in fuel cells is essential for commercialisation of fuel cell systems. Energy is dissipated as heat in particular at the electrode surfaces and in the current conducting materials. Depending on the heat transfer coefficient of the different parts of the fuel cell, we may expect more or less elevated local temperatures inside the cell [1]. Measurements of increased local temperatures along current stream lines have been demonstrated in other electrochemical systems [2].

No research group has, to our knowledge, measured the temperature profile perpendicular to the membrane surface in a polymer fuel cell with Nafion membranes. We report temperature measurements from two layers inside a single fuel cell under operation and measurements of temperatures in the gas channels. The measurements shall be interpreted using equations from irreversible thermodynamics for heterogeneous systems [1]. We argue that the reason for the gradients is a combination of low thermal conductivity of the gas diffusion layers and low heat transfer coefficient of the catalyst surface, combined with high heat production around these layers. The data reduction yields, a first order estimate of the heat transfer coefficient of the electrode active surface, with the assumptions of equal heat transfer coefficients of the electrodes, neglect of Peltier heats and heats of transfer, and a one-dimensional model.

2. The system

The cell is illustrated in Fig. 1. The numbers give the positions of the thermocouples. The numbers can also be used to distinguish between five subsystems: The first subsystem

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The net amount of water transported across the membrane per moles of electrons is different from $\delta_w$ (see below), because some water diffuses from the cathode to the anode.

The side of the membrane facing the anode is denoted (m, a), while the side of the membrane facing the cathode is denoted (m, c).

In subsystem (4), the cathode catalyst surface, protons react with adsorbed oxygen and produce water:

$$H^+(s, c) + \frac{1}{2}O_2(s, c) + e^- (s, c) \rightarrow \frac{1}{2}H_2O(s, c) \quad (6)$$

In subsystem (5) oxygen and water migrate through the porous gas diffusion layer, either in gas phase, or with water in the condensed phase.

$$\frac{1}{4}O_2(g, c) \rightarrow \frac{1}{4}O_2(g, c, 0) \quad (7)$$

$$\frac{1}{2}H_2O(g, c) \rightarrow \frac{1}{2}H_2O(g, c, 0) \quad (8)$$

We shall now make the following assignment of temperatures to positions (cfr. Fig. 1):

$$T_1 = T^{c.a.0}, \quad T_2 = T^s = T^{m.a.}, \quad T_3 = T^{m.c.}, \quad T_4 = T^{g.c.} \quad (9)$$

The first temperature is outside the wall of the porous gas diffusion layer on the anode side, $T^{c.a.0}$. The second temperature is assigned to the anode catalyst surface, $T^{m.a.}$, and is here set equal to the membrane temperature close to the surface, $T^{m.a.}$. The third temperature is assigned to the cathode catalyst surface, $T^{m.c.}$, and the temperature in the membrane close to the cathode catalyst surface, $T^{m.c.}$. The last temperature is the temperature outside the porous gas diffusion layer on the cathode side, $T^{g.c.}$.

Superscripts indicate the location of the variable. The first superscript denotes the bulk phase: s for surface, m for membrane and g for gas phase (backing). The second superscript denotes the anode or cathode side, and the third superscript denotes the position relative to the gas channel. For instance g, a, 0 denotes the gas phase in the backing close to the gas channel.

3. Experimental

All experiments were performed with a fuel cell test-facility designed by Müller-Holst and Vie [9–11]. The electrodes (5 cm²) were home-made. The electrodes were produced with a spraying technique developed by Müller-Holst [9,10]. The electrode slurry was sprayed on the gas diffusion layer (ETEK ELA T, double sided carbon only) to obtain a platinum loading of 0.1 mg/cm², and a Nafion content in the dry electrode of 35 wt.%. The proton-conducting membranes (Nafion 115 and 112) were prepared as described by Müller-Holst [9], and combined with home-made electrodes (Nafion 115) and with
Fuel cell housing was heated to approximately 50% of the minimum pressure of around 10 bar. The gas flow-rates were set to give a conversion of the fuel cell housing with a mechanical pressure of around 30 ml/min. The membrane and electrode assembly was compressed in the literature. The Nafion 115 is 127 μm thick.

The membrane and electrode assembly (MEA) was assembled with two thermocouples (wire: type K, AWG 36–120 μm) in a sandwich on each side of the membrane T₂ and T₃, Fig. 1, inside the electrodes. The sandwiching means that the membrane and backing will be compressed more than normal in the regions where the thermocouples are positioned. Additional thermocouples were positioned in the gas channels T₁ and T₄, Fig. 1. Care was taken to insulate the thermocouples from well conducting parts by coating them with Cyclotene 4026-46 (CYCLOTENE Resins from The Dow Chemical Company), see [11] for further details.

The reacting gases, pure hydrogen and oxygen, were humidified at 80°C, and the gas pressures were 4.5 bar. The membrane and electrode assembly was compressed in the fuel cell housing with a mechanical pressure of around 10 bar. The gas flow-rates were set to give a conversion of the gases of approximately 50%, otherwise the minimum gas flow was 30 ml/min. The fuel cell housing was heated in order to give a temperature in the gas channels of 64°C at open circuit.

The temperatures reported were measured across the membrane-electrode assembly, after a stationary state was reached for a series of current densities. All four temperatures were measured simultaneously as a function of the current density. To establish a quasi-stationary state it took typically between 2–4 min. A typical result for the measurements is presented in Fig. 2.

The figure shows the effect on the temperature and current density by varying the voltage in steps, here from 0.63 to 0.54 V; then to open circuit potential (≈1 V) and finally back to 0.54 V. The corresponding current density increased from approximately 0.5 to 0.68 A/cm², then to 0 A/cm² and back to 0.67 A/cm². All temperatures increased with increasing current density.

At open circuit potential conditions (j = 0), the temperatures were stable around 65°C. The graph shows that stationary states were achieved also after changing the potential. Similar results, as given above, were obtained when commercial electrodes were combined with the Nafion 112 membrane (not shown).

The temperature at the anode catalyst surface (T₂) increased slightly more than the temperature at the cathode catalyst surface (T₃), while the temperatures in the channels stayed significantly lower. The thermocouples we used were large (more than 200 μm thick) compared to the thickness of the electroactive layer (10 μm). This means that the membrane and backing are extra compressed where the thermocouples are positioned, and that the positions of the thermocouples are not well defined. The size of the thermocouples will thus affect the measurements, but in a systematic manner. The temperatures that are measured must be smaller than what should be expected for infinitely thin thermocouples. The thermocouples close to the membrane (T₂ and T₃) block some of the active surface area. Consequently, less heat than normal, will be produced from the fuel cell reaction close to these thermocouples. Also, the thermocouple insulation makes the recorded value lower than the real temperature. So, the reported temperatures must be lower than temperatures of an unperturbed system. The reported temperatures are therefore minimum temperatures. We chose anyhow to report them as average temperatures with standard deviation, determined at a quasi-stationary state. The results for the heat transfer coefficient and thermal conductivities will accordingly be maximum values. Correlations between measured temperatures and current densities were obtained for these conditions.

4.2. The temperature variation

The correlations between temperature and current density were found by varying the cell voltage in different patterns and voltage jumps, and repeating several of the measurements, as described above. In Fig. 3 the measured temperatures were correlated with the corresponding current density. The model for the fit was a 2nd order polynomial:

$$T_j = a_0 + b_1 \cdot j + c_1 \cdot j^2$$  \hspace{1cm} (10) $$

This model fitted the measured temperatures quite well, see Table 1 for the model coefficients and statistical data. The standard deviation varied between 0.5 and 0.7°C. The correlation coefficient was slightly higher for T₂, T₃ than for T₁, T₄. The temperatures in the channels may vary more because the saturated vapor may condense and evaporate, and explain the difference in correlation coefficients.
ture curves in Fig. 3 is shown in Fig. 4. The curve is fairly
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Fig. 3. Curves for the fitted overpotentials at the anode (\(\eta_a\)) and cathode (\(\eta_c\)) and the total resistance (\(r\)) are included.

smooth with little scatter. This confirms that the conditions in the fuel cell were similar for each measured current density and that stationary states indeed were obtained (in agreement with the results illustrated in Fig. 2). The data were fitted to an empirical model (Eq. (11)) based on the Butler-Volmer equation for the overpotentials (Eqs. (12) and (13)). The fit was further described by Vie [11].

\[
E = E^{rev} - (\eta_e + \eta_a) - \eta_{el} \]

(11)

\[
j = \beta_h [\alpha_e^{(rev)/RT} - e^{-(\eta_e^{(rev)/RT})}] \]

(12)

\[
j = \beta_o [\alpha_e^{(rev)/RT} - e^{-(\eta_o^{(rev)/RT})}] \]

(13)

Data in the range 0–0.75 A/cm\(^2\) were included in the model-fit. Values for the exchange current densities at the anode and cathode, and total fuel cell resistance were estimated to: \(\eta^{0}_{a} = 2.5 \times 10^{-7} \text{ A/cm}^2\), \(\eta^{0}_{o} = 4.2 \times 10^{-3} \text{ A/cm}^2\) and \(r_{tot} = 0.31 \Omega \text{cm}^2\). These values described the overpotential curves in Fig. 4, with a standard deviation for the fitted curve of 0.024 V. They refer to the gas composition used here, pure hydrogen and pure oxygen, respectively, both saturated with water. At higher current densities, we observed a steeper drop in fuel cell voltage. A drier membrane or mass transport limitations may explain this. This regime was not included in the calculations.

4.3. Calculations

The measured temperatures and the estimated overpotentials were introduced into the one-dimensional model for the polymer fuel cell given in Appendix A. The water balance over the membrane was solved by iteration in a similar manner as done by Springer et al. [6]. The equations were solved with the data reported above and input data (see next section). As unknowns were taken the heat transfer coefficients of the catalyst surfaces. The heat transfer coefficients were taken to be the same at the anode and cathode, in lack of better knowledge. In a test of the model, we also took the thermal conductivity of the membrane as unknown.

The average thermal conductivity for the gas diffusion layer and the catalyst surface combined, \(K^{av}\), was calculated from Fourier’s law with \(T_1\) and \(T_3\) as boundary conditions.

4.4. Data

Thermodynamic data were taken from standard thermochemical tables [12,13]. The porosity, tortuosity and thickness of the carbon backing, 0.4, 7 and 180 \(\mu\text{m}\), respectively [8], were used to calculate the fluxes in subsystems 1 and 5. The thickness of the Nafion 115 membrane was 127 \(\mu\text{m}\). The density of the membrane in the dry state was \(1.64 \times 10^3 \text{ kg/m}^3\), and the molar weight of Nafion was 1.1 kg mol\(^{-1}\). The diffusion coefficient for the membrane and its electric resistivity were calculated from data presented by Springer et al. [6] and Zawodzinski et al. [7]. The
electric resistivity of the backing was estimated from the total resistance of the fuel cell calculated from the polarisation curve and the calculated membrane resistance [11]. The thermal conductivity of the gas diffusion layer (10 W/mK) was the same as used before [1]. The effective Maxwell-Stefan diffusion coefficient was calculated from the formula given by Slattery and Bird [14].

4.5. Thermal properties of the fuel cell

The results of the calculations are given in Table 2. The table gives the heat transfer coefficient of the catalyst surfaces as a function of current density, for two sets of conditions: no evaporation and full evaporation of water on the cathode side. The result was:

\[ \lambda^c = 1000 \pm 300 \text{ W/m}^2\text{K}^{-1} \]  

The model gave as a control, also the thermal conductivity of the membrane. A small positive number was obtained for the condition that no water evaporates at the cathode. This is the most likely condition since the entering gases are oversaturated with water.

Within the accuracy of the measurements and model used to reduce the data (see Appendix A), the values obtained for \( \lambda^c \) in Table 2 did not depend on the current density. This gives some credence to the measurements and model used to derive the values, since theory requires that the heat transfer coefficient does not depend on \( j \). The values calculated at lower current densities contain larger experimental errors and should be given smaller weight.

The manner we varied the potential within the series of measurements, and the way the system responded, exclude that the cell or the thermocouples were shortcircuited. Elevated temperatures in the membrane electrode assembly were reproduced, but a precise localisation of the thermocouples was not possible. This gives a systematic error in the calculations, which cannot be avoided at the present, and led to an unphysical (negative) membrane thermal conductivity for the condition that no water evaporates at the cathode. This is the most likely condition since the entering gases are oversaturated with water.

It is not necessary to have the membrane thermal conductivity as a variable. We may in stead use as input to the model, our estimate for the membrane conductivity, 0.18 W/mK. Given that the temperature measurements are so crude, we do not find a further refined model appropriate. We conclude that our results are compatible with a membrane thermal conductivity in the range 0 to 0.2 W/mK. More refined experiments, may allow for a better resolution, and a more detailed analysis. The fact that maximum values are reported for the thermal conductivities, is a challenge for further work.

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5. Discussion

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The catalyst surface of the electrode.

A first qualitative explanation of the measurements in Fig. 2 is now possible. The rise in anode temperature can be ascribed to water condensing at the anode, while the cathode is heated because the high cathode overpotential is heating the cathode catalyst surface more, than evaporating water is cooling this place. Little water is evaporating from the cathode since the entering cathode gas is oversaturated with water. The results above are compatible with this explanation. The low heat transfer coefficient of the surface means that it is likely that heat accumulates at the electrode surfaces during operation. A temperature gradient like the one we observe, may then follow, given the asymmetry in the overpotentials and in the water transport.

The observations were not unique for the Nafion 115 membrane electrode assembly. We obtained similar results when commercial electrodes were combined with the Nafion 112 membrane (not shown). This is expected. The membrane thermal conductivity should not change by changing from Nafion 115 to Nafion 112, which is thinner. Also, the nature of the adsorbed gas and water layer at the catalyst, should be more important for the heat transfer coefficient, than the type of catalyst material.

6. Conclusion

A temperature gradient perpendicular to the membrane in the single fuel cell has been reported in this work. The local temperature varied as a second order polynomial in the electric current density. From this information, the current density at the catalyst, should be more important for the heat transfer coefficient, than the type of catalyst material.

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Appendix A. Governing equations

We give the equations that were used for calculation of the surface heat transfer coefficient. The set of equations is solved by solving five sets of transport equations for the subsystems plus the conservation equations. The equations are given with reference to the system’s description in Section 2. Numerical values were obtained from the literature as described in Section 4.4. For further details on the use of excess variables for the surface, it is referred to Kjelstrup et al. [1] and Blinov et al. [17]. The terminology follows that of de Groot and Mazur [18]. Super- and subscripts were explained at the end of Section 2.

A.1. Conservation equations

The measurements were done at stationary state conditions. The energy flux as well as certain mass fluxes, are then constant through the system. For the fluxes of hydrogen and oxygen we have

\[ j_{H_2} = \frac{j}{F}, \quad j_{O_2} = j \frac{i}{F} \]

Here \( j \) is the current density and \( F \) is Faraday’s constant. As positive direction of transport, we take the direction from the left to the right in Fig. 1. The water flux through the anode \( j_{H_2}^{w} \) and anode catalyst surface is equal to the net water flux in the membrane.

\[ j_{H_2}^{w} = j_{H_2}^{aq} + j_{H_2}^{g} = j_{H_2}^{aq} + \left( j \frac{i}{F} \right) \]

At the cathode catalyst surface there is formation of water,

\[ j_{H_2}^{c} = \frac{j_{H_2}^{aq}}{2} + \left( j \frac{i}{F} \right) \]

The energy flux \( J_{e} \) is defined as the sum of the physical heat flux \( J_{e}^{p} \), the sum of enthalpy moving with each component \( \sum_{i} \rho_{i} h_{i} \) and the electric energy, \( j \phi \).

\[ J_{e} = J_{e}^{p} + \sum_{i} \rho_{i} h_{i} + j \phi \]

The summation is taken over water and hydrogen (in the anode) or water and oxygen (in the cathode). The electric potential \( \phi \) needs a reference to be well defined. The energy flux is constant everywhere, so \( \partial J_{e}/\partial x = 0 \). We assume that the enthalpies of water, hydrogen and oxygen is constant in the temperature interval that we consider. For the gas diffusion layer as well as for the membrane, we then have

\[ \frac{\partial J_{e}^{p}}{\partial x} = -j \frac{\partial \phi}{\partial x} \]

It is necessary to consider the surface layers as separate thermodynamic systems, in order to understand the measured temperature gradients. The thermal conductivities obtained may be useful in predicting heat fluxes in a stack. The model for heat transfer included only the main transport coefficients. An improved model should include coupling terms in the description, have different conductivities for each surface, and allow for transport in more directions.
The change in the measurable heat flux at a location \( x \) is thus given by the change in electric power in these phases (subsystems \( 1, 3 \) and \( 5 \)).

The energy flux is also constant through the electrode surfaces (subsystems \( 2 \) and \( 4 \)). This means that heat fluxes on each side of a surface depends on the large contributions from enthalpy changes at the surface. For the anode and cathode catalyst surfaces, we have:

\[
\begin{align*}
J_{q, a}^{m,c} - J_{q, s}^{m,c} &= j(\phi^{m,a} - \phi^{m,s}) \\
J_{q, a}^{w,a} &= j(\phi^{w,a} - \phi^{w,s}) \\
J_{q, s}^{w,s} &= j(\phi^{w,s} - \phi^{w,s}) \\
\end{align*}
\]

The equations express that the molar enthalpies are transformed into heat and electrical power at the membrane-electrode surfaces. By introducing the conservation equations for mass and the enthalpy difference \( \Delta_{\text{mem}} H_a = h_a^{m,a} - h_a^{m,s} \), we obtain for the anode catalyst surface,

\[
J_{q, a}^{m,a} - J_{q, s}^{m,a} = -j(\phi^{m,a} - \phi^{m,s}) + h_a^{m,a} \left( \frac{\Delta T_a}{T_a} \right) + j_a \Delta_{\text{mem}} H_a
\]

We shall use the enthalpy of evaporation \( \Delta_{\text{mem}} H_a \) for \( \Delta_{\text{mem}} H_c \), to describe the enthalpy change when water is going from the liquid state in the membrane to the vapour state. The corresponding equation for conservation of energy at the cathode catalyst surface is:

\[
J_{q, c}^{m,c} - J_{q, s}^{m,c} = -j(\phi^{m,c} - \phi^{m,s}) + h_c^{m,c} \left( \frac{\Delta T_c}{T_c} \right) - j_c \Delta_{\text{mem}} H_c
\]

A.2. Subsystem 1: anode gas diffusion backing

In the porous carbon matrix, there is conduction of heat and electric charge. The following flux equations were used:

\[
\begin{align*}
J_{q, a}^{m,a} &= -\lambda_a^{m,a} \frac{\Delta T_a}{dx} \\
\frac{\partial \phi^{m,a}}{\partial x} &= -\sigma^a j \\
\end{align*}
\]

where \( \sigma^a \) is the electric resistivity of the carbon matrix. We see from the energy balance (Eq (19)), that heat is generated.

Eq. (23) describes how it is conducted away, in the absence of Poeltier effects. We shall assume that \( J_{q, a}^{m,a} \) is constant in the porous backing.

Diffusion of hydrogen and water was modelled with Maxwell-Stefan’s equation, as is common in these systems:

\[
\frac{d \rho_i}{dx} = \frac{RT}{D_i} (x_i J_i - x_j J_j)
\]

Here \( p_i \) is the partial pressure of component \( i \), and \( D_i^{ij} \) is the Maxwell-Stefan interdiffusion coefficient. The flux equations on the cathode side are similar to these.

A.3. Subsystem 2: anode catalyst surface

The simplified equations for the anode catalyst surface were (see [1] for more details):

\[
\begin{align*}
J_{q, a}^{m,a} &= -\lambda_a^{m,a} \frac{\Delta T_a}{dx} \\
(\phi^{m,a} - \phi^{m,s}) &= \eta^a
\end{align*}
\]

The electric potential jump across the surface, \( \phi^{m,a} - \phi^{m,s} \), was taken as the overpotential, \( \eta^a \), of this electrode, since the reversible potential of the hydrogen electrode is zero by definition. The Fourier heat transfer coefficient of the surface, \( \lambda_a^{m,a} \), has the dimension of a heat transfer coefficient, \( \text{W/m}^2\text{K} \), and shall for this reason be called so, to distinguish it from the thermal conductivities of the bulk phases. The heat flux into the surface from the porous gas diffusion layer, \( J_{q, a}^{m,a} \) (see also Eq. (20)), is governed by \( \lambda_a^{m,a} \). There is also a heat flux out of the surface into the membrane, \( J_{q, a}^{w,a} \). This heat flux is calculated here from energy balance for the surface. We have assumed equilibrium for water across the surface.

A.4. Subsystem 3: membrane

In the membrane, water is driven by two forces, the electric force and the chemical potential of water. In agreement with others [6], the flux equations in the membrane can be written (see [1] for more details):

\[
\begin{align*}
J_{q, m}^m &= -\lambda_m^{m} \frac{\Delta T_m}{dx} \\
\frac{\partial \rho_m}{\partial x} &= -D_m \frac{\partial u_m}{\partial x} + \rho_m \frac{\partial}{\partial x} (\frac{1}{F}) \\
\end{align*}
\]

The coefficient \( \lambda_m \) is the thermal conductivity of the membrane, \( D_m \) is the membrane water diffusion coefficient, \( \rho_m \) is the water transference coefficient, and \( \eta_m \) is the electrical resistivity of the membrane phase. Here \( J_{q, m}^m \) is the measurable heat flux in the membrane and \( F_{q, m} \) is the water flux in the membrane, and \( \frac{d \rho_m}{dx} ) \) is the gradient in electric potential in the membrane.

A.5. Subsystem 4: cathode catalyst surface

The flux equations for the cathode catalyst surface are similar to those of the anode catalyst surface (subsystem 2):

\[
\begin{align*}
J_{q, c}^{m,c} &= -\lambda_c^{m,c} \frac{\Delta T_c}{dx} \\
(\phi^{m,c} - \phi^{m,s}) &= -\frac{1}{F} \left( \frac{1}{2} \lambda_c^{m,c} \frac{\partial u_c}{\partial x} + \frac{1}{2} \lambda^{m,c} \frac{\partial u_m}{\partial x} \right) + \eta^c
\end{align*}
\]
The first equation gives the heat flux in the absence of Peltier and Soret effects. The overpotential of the electrode is \( \eta \). The second equation expresses that the surface potential jump is caused by the chemical reaction of hydrogen and oxygen minus the overpotential.

The surface heat transfer coefficient for the cathode catalyst surface may be different from that of the anode catalyst surface, since there is more water present at the cathode. We have no good argument for how different the coefficients should be, so we shall use the assumption that they are equal:

\[
\lambda_{sc} = \lambda_{sa} = \lambda_s
\]  

(A.33)

A.6. Subsystem 5: Cathode gas diffusion backing

The equations for this subsystem are the same as for subsystem 1.

A.7. Consistency of the model

With the above equations, one can verify that sum of all potential drops across all subsystems is the familiar expression for the cell potential, Eq. (11). The total electric resistance of the cell is then:

\[
r = r_{ad}^a + r_{md}^m + r_{cd}^c
\]  

(34)

The resistances of the surfaces were included in the overpotentials.

Appendix B. List of symbols

B.1. Latin characters

- \( d_y \) thickness of phase \( y \)
- \( D_w \) membrane water diffusion coefficient
- \( D_y^{\text{eff}} \) Maxwell-Stefan’s interdiffusion coefficient
- \( F \) Faraday constant (96,485 C/mol)
- \( h_x \) enthalpy for substance \( x \) (J/mol)
- \( j_{0x} \) exchange current density (A/cm\(^2\))
- \( J_y \) flux of substance \( x \) in part \( y \) (mol/(m\(^2\)s))
- \( J_q \) heat flux (J/m\(^2\)s)
- \( p_i \) partial pressure of substance \( x \) (Pa)
- \( r_y \) resistance in phase \( y \) (\( \Omega \m)\)
- \( s_y \) correlation coefficient
- \( R \) the universal gas constant (8.3143 J/Kmol)
- \( s_{x_k} \) standard deviation for model \( i \)
- \( s_{y}^{\text{eff}} \) transference coefficient of water in membrane
- \( T_y \) temperature in part \( y \) (K)
- \( s_{x} \) mole fraction of substance \( x \)

B.2. Greek letters

- \( \alpha \) symmetry factor in Butler-Volmer equation
- \( \eta \) overpotential
- \( \lambda_m \) thermal conductivity in membrane (W/mK)
- \( \lambda_w \) thermal conductivity in the catalyst surface and gas diffusion layer (W/mK)
- \( \lambda_s \) heat transfer coefficient of catalyst surface (W/m\(^2\)K)
- \( \mu_x \) chemical potential (J/mol)
- \( \phi \) electric potential (V)

B.3. Subscripts

- \( e \) energy
- \( H_2 \) hydrogen
- \( O_2 \) oxygen
- \( H_2O \) water
- \( q \) heat
- \( \text{tot} \) total
- \( w \) water

B.4. Superscripts

- \( 0 \) position close to gas channel
- \( a \) anode
- \( b \) backing
- \( c \) cathode
- \( g \) gas
- \( m \) membrane
- \( s \) surface
- \( \text{saturated} \) saturated
- \( \text{tot} \) total
- \( \text{vap} \) vapour

References


